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sical faults or defects of the surface are the only ones which subsist.

"But, guided by the observation itself which causes them to be discovered, one succeeds, as I have demonstrated before, in correcting locally these defects; and lastly they can be reduced to such proportions as to exercise no sensible influence on the quality of the images.

"The mechanical processes by which the working of glass surfaces is usually effected seem to lose of their efficacy as soon as they are applied to pieces of unusual dimensions; the results which they then furnish are but an approximation, a sort of trial which is far from being satisfactory; but where mechanism becomes powerless, the hand of man can do something more: assisted by the resources which optics put at his disposal, and guided by a system of observations the power of which increases with that of the instrument which is to be constructed, the human hand is then enabled to proceed with the work, and to carry it out to the greatest degree of precision.

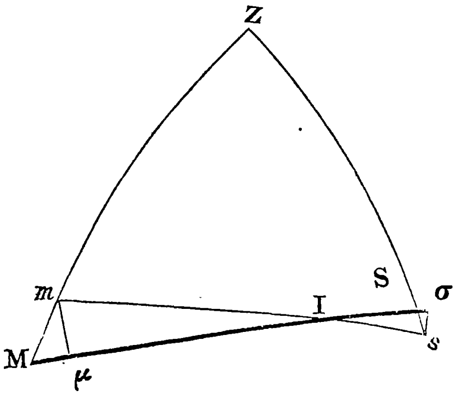
"If it be required to verify by optical means any one of the ellipsoidal surfaces which lead from the sphere to the paraboloid, the same process can be applied identically, on condition, that the object which is used as a mark is placed at one of the foci, and the image examined at the other focus. The information which is got thereby, and the corrections which must be deduced therefrom, are based on the same appearances; only they have reference especially to the sort of surface given by the position of the foci, and the inequalities which are found out show deviations which are reckoned in the same way as for the spherical figure, viz., from the level of the correct surface which it is desired to obtain."

On a New Method of Clearing Lunars.

By Lieut.-Col. R. Shortrede.

The following method of clearing a lunar distance gives a result always true to within a few hundredths of a second, and, requiring few logarithms, and these too from five to three places, is shorter and easier than any other I am acquainted with:—

m, M, and s, S, being the true and apparent places of the moon and star, and perpendiculars $m\,\mu$, $s\,\sigma$, being drawn to M S, the apparent distance $m s = \mathrm{M\,S} - \mathrm{M}\mu + \mathrm{S}\sigma + \mathrm{I}m - \mathrm{I}\mu + \mathrm{I}s - \mathrm{I}\sigma$.



For the principal corrections $\mathrm{M}\mu$, $\mathrm{S}\sigma$, the proper expressions are

$$\tan \mathrm{M}\mu = \tan \mathrm{M}m \cos \mathrm{M}$$

and $$\tan \mathrm{S}\sigma = \tan \mathrm{S}s \cos \mathrm{S}$$

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but as the triangles differ little from plane ones, we may without much error assume

$$\mathrm{M}\mu = \mathrm{M}m \cos \mathrm{M} = \mathrm{M}m - \mathrm{M}m\ 2 \sin^2 \tfrac{1}{2}\mathrm{M},$$

and $$\mathrm{S}\sigma = \mathrm{S}s \cos \mathrm{S} = \mathrm{S}s - \mathrm{S}s\ .\ 2 \sin^2 \tfrac{1}{2}\mathrm{S}.$$

Applying these corrections we have the curtate or reduced distance $\mu\sigma$.

With a given base $\mathrm{I}\mu$, the excess $\mathrm{I}m - \mathrm{I}\mu$ is as μm^2, and as μm is not directly given, it is usual to take $\mathrm{M}m^2 - \mathrm{M}\mu^2 = \mu m^2$; and then, by means of a small table, to take the difference of the corrections for $\mathrm{M}m$ and for $\mathrm{M}\mu$.

When the lunar distance is small and the corrections considerable, the tabular quantities vary rapidly; a small table of double entry is then inconvenient; and if the table be extended to include all cases, it becomes inconveniently large, and instead of finding the corrections for $\mathrm{I}m$ and $\mathrm{I}s$ separately, as they ought to be, it is usual to make one general correction for MS; the result so found may be in error by 4″ or 5″.

Legendre and Delambre, using a different method with the angle I, gave formulæ for finding the secondary correction; but as Delambre admits that his process of calculation is long, and that Legendre's process, though, as regards the logarithmic work, shorter than his own, requires a long and tedious preparation.

In the method here proposed we have to find $\mathrm{I}s$, and then the secondary corrections.

To find $\mathrm{I}s$ we have

$$\mathrm{Sin\ I} = \frac{\sin \mathrm{S}s\ .\ \sin \mathrm{S}}{\sin \mathrm{I}s} = \frac{\sin \mathrm{M}m\ .\ \sin \mathrm{M}}{\sin \mathrm{I}m}$$

$$\therefore\ \sin \mathrm{I}s = \frac{\sin \mathrm{S}s}{\sin \mathrm{M}m} \cdot \frac{\sin \mathrm{S}}{\sin \mathrm{M}} \sin \mathrm{I}m = \frac{\sin \mathrm{S}s}{\sin \mathrm{M}m} \cdot \frac{\sin \mathrm{ZM}}{\sin \mathrm{Z}s} \cdot \sin \mathrm{I}m$$

$$= \frac{\mathrm{S}s}{\mathrm{M}m}\ \frac{\cos \mathrm{Alt.}\ \text{☽}}{\cos \mathrm{Alt.}\ *} \sin \mathrm{I}m$$

$\mathrm{I}m$ is properly the difference between $\mathrm{I}s$ and the corrected distance sm. Instead of $\mathrm{I}m$ we may, for a first approximation, use the apparent distance SM, or the reduced distance $\sigma\mu$, and then subtracting the value thus found from $\sigma\mu$, we may, by repeating the process, find the true value of $\mathrm{I}s$ as near as we please. This is abundantly easy as logarithms, to 3 places, are sufficient, all of which, excepting that for the corrected value of $\mathrm{I}\sigma$, are already given in the calculation of the principal corrections.

The triangle $\mathrm{I}\mu m$ being right-angled, putting $\mu m = p$ for the difference d between the hypothenuse h and the base b, we have the expression $\sin d = 2 \sin \frac{p-d}{2}\ .\ \sin \frac{p+d}{2}\ ..\ \cot b$, which, as p is always less than 1° and d less than 1′, may con-

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veniently be reduced to $d'' = \frac{\sin 1''}{2} . p''^2 . \cot b$: for $p^2 = \mu m^2$ we have $(Mm - M\mu)(Mm + M\mu)$, whereof the first term is given in finding the principal correction, and the second term by adding together the parts already found, $\log \frac{\sin 1''}{2} = 4{\cdot}38454$ or simply 4·385; so that log cot $I\mu$ is the only trigonometrical term requiring to be found. It may be remarked that Is being always a small arc, its cotangent will differ little from its cosecant or the arithmetical complement of log sin Is.

For the moon's correction logs to 5 places should be used, and for the star correction 3 or 4 places may be used according as the star may be high or low.

All these corrections are easily found true to a hundredth of a second. If extreme precision were required, a small correction $= -\frac{Mm^3}{3R^2} \cos M \sin^2 M$ should be applied on account of the difference between arc and tangent in the triangle $Mm\mu$.

The true expression $\tan M\mu = \tan Mm \cos M$ reduced to arc gives $M\mu + \frac{M\mu^3}{3R^2} = Mm^3 \cos M + \frac{Mm^3}{3R^2} \cos M \therefore M\mu = Mm \cos M + \frac{Mm^3}{3R^2} \cos M - \frac{M\mu^3}{3R^2}$; the principal term being subtractive, the spherical correction becomes $-\frac{Mm^3}{3R^2} \cos M + \frac{Mm^3}{3R^2} \cos^3 M = -\frac{Mm^3}{3R^2} \cos M \sin^2 M$.

The greatest possible value of this correction can never exceed eleven hundredths of a second.

Note on the recent Occultation of Saturn by the Moon, and on Experiments for ascertaining the Polarisation of the Moon's Light. By Professor Secchi.

(*Extract of a Letter to Admiral Manners, dated Rome, May* 9, 1859).

"We observed the occultation of *Saturn* on Sunday last with a very fine evening and steady image. We used for the purpose two different telescopes, the large refractor, power 600, and a smaller $3\frac{1}{2}$-inch Fraunhofer, but we could not observe any distortion at all attributable to a lunar atmosphere. At the immersion the dark limb of the moon was very visible and distinct, so that any variation in the form of the limb of the planet would be very sensible. I noticed only the irregular outline of the limb of the moon, the planet happening to disappear between two mountains. In consequence of this circumstance the last segment of the ring, as observed at the immersion, acquired the form of an oblong bead. At the reappearance the planet was so faint in comparison with the luminous edge of the moon that it could scarcely be perceived until it had already emerged a little. It looked like *Saturn*